Estimation of the degradation rates of a system with preventive maintenance Renewal theory approach

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Overview

Framework

The renewal approach

Generalizations and Perspectives

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Generalizations and Perspectives

Setting

- We observe a park of complex systems
- We focus on a special component of this system
- This particular component appears several times in each system
  - In the same conditions
  - The number of observations is not so bad
- The park is submitted to a maintenance
  - Periodic inspections are operated (c is the period)
- Our component can be in three states
  - Sane (S): It works perfectly (revealed by an inspection)
  - Damaged (D): It needs to be repaired (revealed by an inspection)
  - Down (F): Out of order (revealed when the failure occurs)

Transition Times

- We model the transit times between the states as follows :
  - ⋆ Y^s is the time spent in the state (S) : $Y^s \sim \mathcal{E}(\mu)$
  - ⋆ Y^d is the time spent in the state (D) : $Y^d \sim \mathcal{E}(\lambda)$

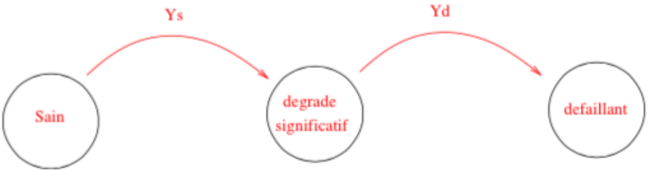


Figure: Transition Times

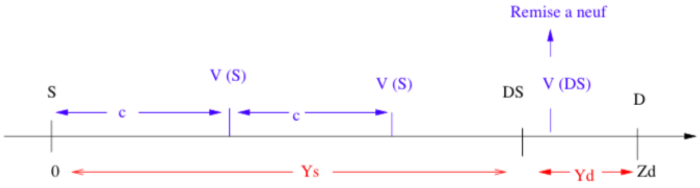
We want to estimate the unknown transition rates μ and λ

The inspections

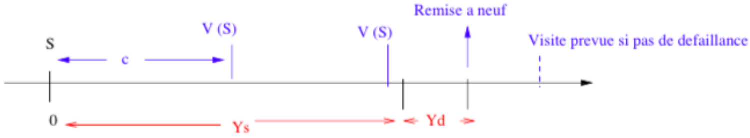
- Let C_i the time between the inspections $\#i-1$ and $\#i$ ($C_i = c$ for the moment)
    - $D_i = C_1 + \ldots + C_i$ the date of the inspection $\#i$
- During an inspection
    - if the component is safe, nothing is done
    - if it is damaged, the component is immediately repaired and returns to the state (S)
- If a failure occurs, the component is also repaired and returns to the state (S)

Two kinds of repairs

- An inspection reveals that the component is damaged



- A failure occurs



Notations

- We denote

$$\begin{aligned} D_{K^r} &= \text{date of the planed inspection following the damaged state} \\ K^r &= \text{the corresponding index} \\ &= \text{number of inspections between two repairs} \\ &= \inf\{n \geq 1 : D_n \geq Y^s\} = 1 + \sum_{n \geq 1} \mathbf{1}_{D_n < Y^s} \\ N_t^r &= \text{the number of repairs up to time } t \\ N_t^f &= \text{the number of failures up to time } t \\ N_t^i &= \text{the number of inspections up to time } t \end{aligned}$$

⋆ In case of periodic inspections

$$K^r = \lceil Y^s / c \rceil , \qquad D_{K^r} = c \lceil Y^s / c \rceil$$

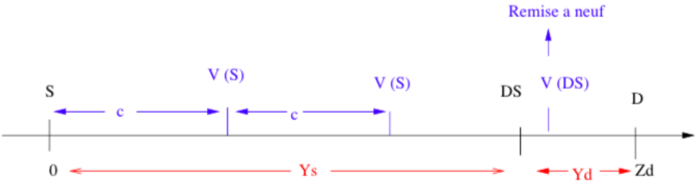
Focus on a single element

After a repair, the component is considered as new

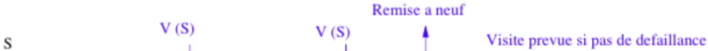
N^r is a renewal process with inter-repair time X^r with

$$X^r = \min(D_{K^r}, Y^s + Y^d)$$

- ⋆ $X^r = D_{K^r}$: the repairs is due to a planed visit with damaged state



- ⋆ $X^r = Y^s + Y^d$: a failure occurs



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The data

- The data contains
  - ⋆ The ID of the component
  - ⋆ The inspection dates and the state of the component
  - ⋆ The failure dates

ID	Date of the event	State
1	01/01/1995	Inspection: OK
1	01/06/1995	Inspection: OK
1	01/11/1995	Inspection: repair
1	01/01/1996	Inspection: repair
2	01/01/1995	Inspection: OK
2	15/04/1995	Failure: repair

The main point of the method

- For each component (e.g. #1), the number of repairs, N^r, is a renewal process

Main arguments of the approach

- We have several independent components observed on rather short times
- We aggregate data and consider the result as a single realization observed on a long time
- We can use the asymptotic behavior of renewal processes
  - ⋆ Law of large numbers
  - ⋆ Central limit theorem

Basic observations

- N^r is a renewal process with inter-arrival time

$$X^r = \min(D_{K^r}, Y^s + Y^d)$$

- N^f is also a renewal process with inter-arrival time

$$X^f \overset{(d)}{=} \sum_{j=1}^{\tau} X_j^r, \qquad \tau = \inf\{n \geq 1 : D_{K_n^r} \geq Y_n^s + Y_n^d\}$$

  - ⋆ τ is a geometric time with parameter $P_d = \mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right)$

- N^i is almost a renewal process with rewards
  - ⋆ Between two repairs, the number of inspections is K^r

$$N_t^i = \sum_{j=1}^{N_t^r} K_j^r + R_t, \quad R_t = \#\text{inspections between the last repair and } t$$

Asymptotic behavior

Law of Large Numbers

$$\lim_{t\to\infty} \frac{N_t^i}{t} = \frac{\mathbb{E}[K^r]}{\mathbb{E}[X^r]} \quad \text{depends only on } \mu \ ,$$

$$\lim_{t\to\infty} \frac{N_t^f}{t} = \frac{\mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right)}{\mathbb{E}[X^r]}, \qquad \lim_{t\to\infty} \frac{N_t^r}{t} = \frac{1}{\mathbb{E}[X^r]}$$

- When $Y^s \sim \mathcal{E}(\mu)$ and $Y^d \sim \mathcal{E}(\lambda)$

$$\mathbb{E}\left[X^r\right] = \frac{1}{\mu} + \frac{1}{\lambda}\mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right) \quad \mathbb{E}\left[K^r\right] = \frac{1}{1 - e^{-\mu c}},$$

$$\mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right) = 1 - \frac{\mu}{\mu - \lambda}\frac{e^{-\lambda c} - e^{-\mu c}}{1 - e^{-\mu c}}$$

Very simple estimators

- Combining these limits (in particular)

$$\lim_{t\to\infty} \frac{N_t^i}{N_t^r} = \mathbb{E}\left[K^r\right], \quad \lim_{t\to\infty} \frac{N_t^f}{N_t^r} = \mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right)$$

- We get very simple estimators

$$\mu = \lim_{t\to+\infty} -\frac{1}{c}\log\left(1 - \frac{N_t^r}{N_t^i}\right) = \lim_{t\to+\infty} \widehat{\mu}_t,$$

$$\lambda = \lim_{t\to+\infty} \frac{-N_t^f \log\left(1 - \frac{N_t^r}{N_t^i}\right)}{-t\log\left(1 - \frac{N_t^r}{N_t^i}\right) - cN_t^r} = \lim_{t\to+\infty} \widehat{\lambda}_t$$

  - ⋆ In particular, if $N_t^r << N_t^i$,

$$\widehat{\mu}_t = \frac{N_t^r}{cN_t^i}, \qquad \widehat{\mu}_t = \frac{N_t^f}{t - cN_t^i}.$$

Central Limit Theorem

- With the help of Rényi's theorem, we can prove the following result

Theorem

$$\sqrt{t}\left(\frac{N_t^i}{t}-\frac{\mathbb{E}[K^r]}{\mathbb{E}[X^r]}, \frac{N_t^f}{t}-\frac{\mathbb{P}\left(D_{K^r}\geq Y^s+Y^d\right)}{\mathbb{E}[X^r]}, \frac{N_t^r}{t}-\frac{1}{\mathbb{E}[X^r]}\right) \xrightarrow{(d)} \mathcal{N}(0,Q)$$

where Q is explicit.

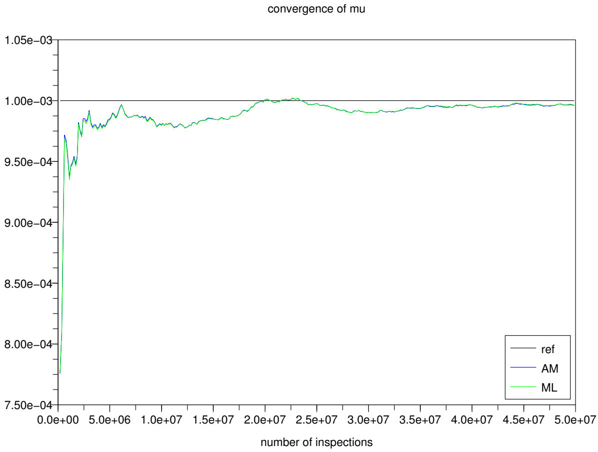
Corollary (Asymptotic Normality)

$$\sqrt{t}\left(\widehat{\mu}_t-\mu, \widehat{\lambda}_t-\lambda\right) \xrightarrow{(d)} \mathcal{N}(0,R)$$

- This gives confidence intervals

Simulated data

- $\mu = 10^{-3}$, $\lambda = 5.10^{-4}$, $c = 1000$, $t = 50001908$
  - $N_t^r = 33501$, $N_t^f 8255$, $N_t^i 53116$



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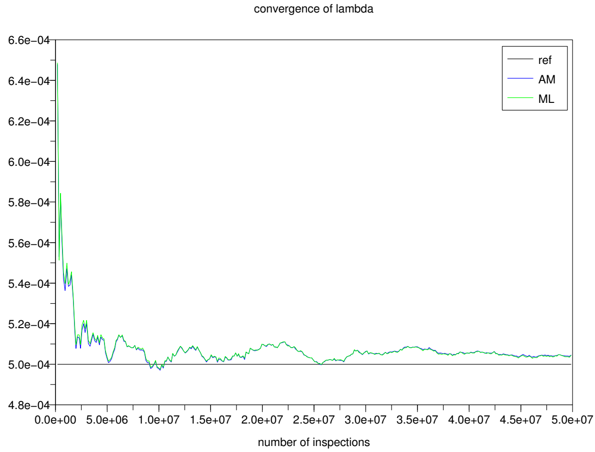

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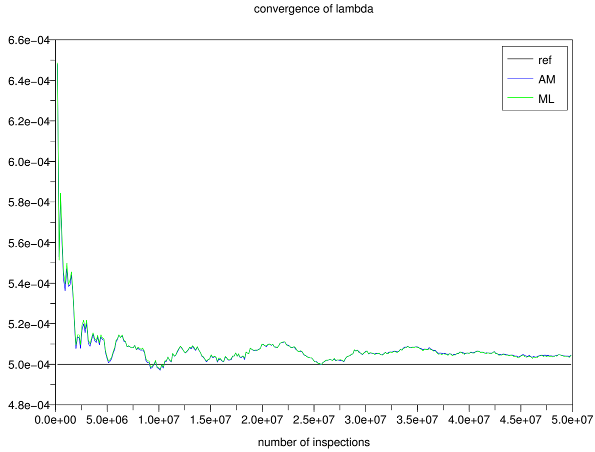

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Method	μ and CI ($\times 10^{-4}$)	λ and CI ($\times 10^{-4}$)
ML	9.96215 (0.056)	5.04164 (0.056)
AM	9.96184 (0.054)	5.04197 (0.053)

- Computational times
  - ⋆ ML = 62 s.
  - ⋆ AM = 10^{-4} s.

Real data

- $N_t^r = 51$, $N_t^f = 5$, $N_t^i = 157$

Mthod	$\widehat{\mu}$	$\widehat{\lambda}$	$\log V$
ML	$9.44\,10^{-6}$	$2.21\,10^{-6}$	-146.59
MA	$9.60\,10^{-6}$	$4.41\,10^{-6}$	NA

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Random inspections

- We can add some randomness on the inspection dates
  - $\star$ C_i i.i.d. with a known law: $\mathcal{L}(C)$
- Roughly speaking, it changes $e^{-\mu c}$ by $L(\mu)$ where

$$L(s) = \mathbb{E}\left[e^{-sC}\right]$$

$$\mathbb{E}\left[X^r\right] = \frac{1}{\mu} + \frac{1}{\lambda}\mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right),$$

$$\mathbb{E}\left[K^r\right] = \frac{1}{1 - L(\mu)},$$

$$\mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right) = 1 - \frac{\mu}{\mu - \lambda}\,\frac{L(\lambda) - L(\mu)}{1 - L(\mu)}$$

Extensions

- Closed formulae for $Y^s \sim \gamma(n, \mu)$, $n \in \mathbf{N}$.
- For $n = 2$, we have

$$\mathbb{E}\left[K^r\right] = \frac{1 - L(\mu) - \mu L'(\mu)}{(1 - L(\mu))^2},$$

$$1 - P_d = \frac{\mu^2}{(\mu - \lambda)^2}\left(\frac{L(\lambda) - L(\mu)}{1 - L(\mu)} + (\mu - \lambda)L'(\mu)\frac{1 - L(\lambda)}{(1 - L(\mu))^2}\right).$$

- Four states model
- External factors e.g. weather

What can we hope from this method?

- We have **3** equations

$$\lim_{t\to\infty} \frac{N_t^i}{t} = \frac{\mathbb{E}[K^r]}{\mathbb{E}[X^r]}$$
$$\lim_{t\to\infty} \frac{N_t^f}{t} = \frac{\mathbb{P}\left(D_{K^r} \geq Y^s + Y^d\right)}{\mathbb{E}[X^r]},$$
$$\lim_{t\to\infty} \frac{N_t^r}{t} = \frac{1}{\mathbb{E}[X^r]}$$

- We can estimate **3** three parameters

  ⋆ An exponential law for Y^d seems reasonable $\mathcal{E}(\lambda)$

  ⋆ This method should work for the law of Y^s depending on two parameters

  ⋆ e.g. $Y^s \sim \mathcal{W}(\beta, \eta)$ which seems to be very popular

Weibull case or another law

- In the exponential case, we have simple closed formula
- The first idea was to do a lot of computations to get formulae
  - ⋆ semi-analytical formulae
  - ⋆ expressed with the help of series, integrals
- Drawbacks
  - ⋆ Not really explicit in the end
  - ⋆ You have to do all the computations each time you consider another law

Monte-Carlo approach

- We are trying to couple the renewal estimation with Monte-Carlo simulations
- The renewal approach in two equations

$$\lim_{t \to +\infty} t^{-1} \left(N_t^i, N_t^f, N_t^r \right) = F(\text{parameters}),$$

$$\widehat{parameters} = F^{-1} \left(t^{-1} \left(N_t^i, N_t^f, N_t^r \right) \right)$$

Monte-Carlo approach

- In the exponential case, the function F is explicit

For the general case

Compute F using Monte-Carlo simulations

- Method in development
- Reduction of the computational cost

Weibull case

- $Y^s \sim \mathcal{W}(\beta, \eta)$, $Y^d = \mathcal{E}(\lambda)$, $C \sim \mathcal{N}(c, \sigma^2)$
- $\beta = 1.5$, $\eta = 500$, $\lambda = 510^{-4}$
  - ⋆ $c = 1000$, $\sigma^2 = 10$
- Three methods (100 times)
  - ⋆ A grid method: 6879 s.
  - ⋆ Optim 1: 52.47 s.
  - ⋆ Optim 2: 264 s.

Weibull case

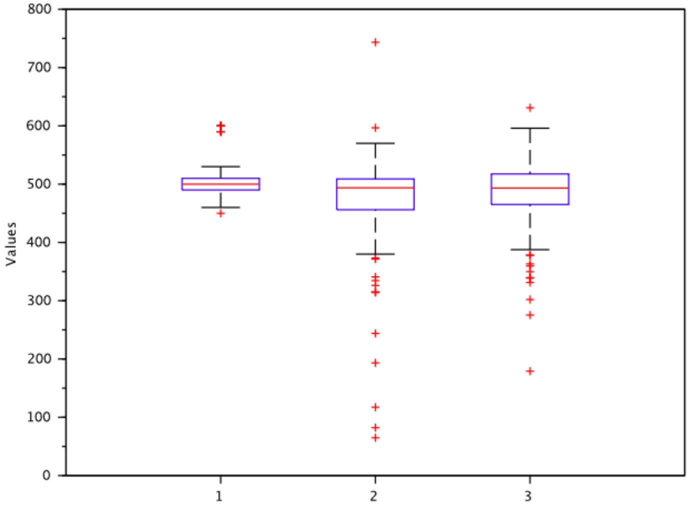

Figure: η

Weibull case

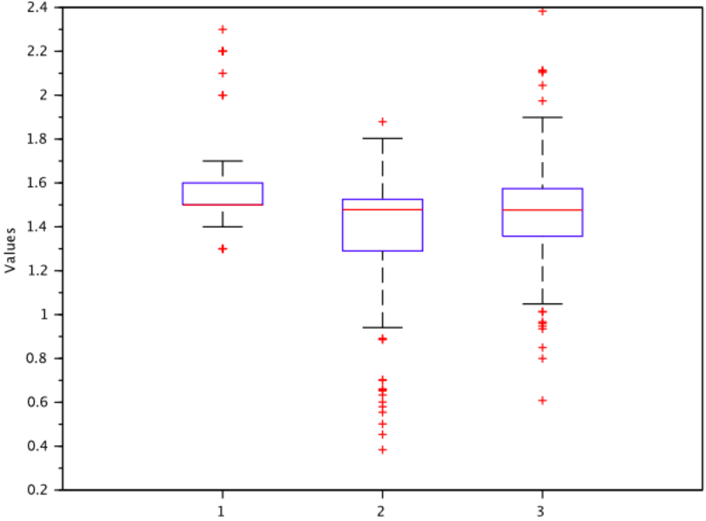

Figure: β

Weibull case

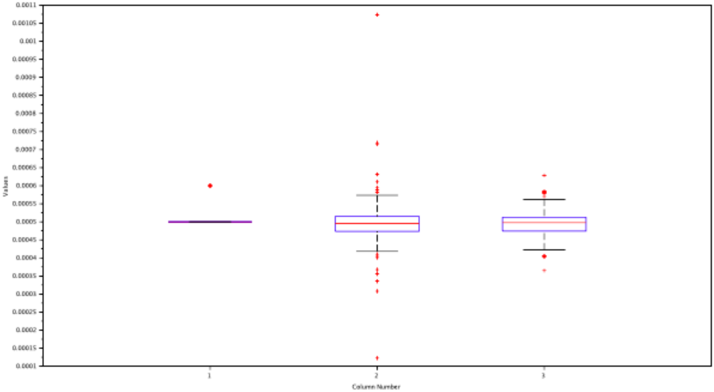

Figure: λ

Monte-Carlo approach

- Still numerical tests to do
- Seems to work so far

Thank you for your attention